Dynamic Conditional Correlations and Tail Dependencies: Copula-Based Vertical Price Co-movement Analysis

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Abstract

This study extends the existing application to a generalized autoregressive score (GAS) framework of Creal et al. (2013), which allows the price dependence to exhibit a dynamic feature. We derived the dynamic conditional correlations and time-varying tail dependencies, which enabled us to evaluate market co-adjustments for specific periods under changing and complex circumstances. Empirical application is to the farm-retail price co-movements of the U.S. pork industry, which has undergone substantial changes in the past four decades. The results confirmed the existence of asymmetry between positive and negative extreme (tail) co-movements. Furthermore, results showed that price adjustments were highly linked in the 1970s and 1980s; however, the degree of dependence dwindled in the mid-1980s, reaching historical lows around the hog crisis in 1998/1999, and has remained low since then. One important implication is that consumers have not benefited as much as expected from the increased productivity resulting from technological and organizational improvements because retailers do not pass on the associated cost reductions.

Keywords: vertical price transmission, time-varying copula, dynamic conditional correlation, tail dependence, pork industry

JEL codes: C32, Q11

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1. Introduction

Vertical price transmission links input prices to output prices and often reveals the extent to which retail commodity markets respond to changes at the raw material level. The degree to which market shocks are transmitted up and down the marketing chain has long been considered an important indicator of market performance. Much of the motivation underlying this line of research has involved concerns about market power and the potential effects that increased market concentration may have on producer and consumer welfare.

A wide variety of empirical research has focused on the asymmetry of price transmission or co-movements. Early work usually divided price changes in input level into two groups conditioned on the direction or magnitude of changes and then investigated transmission coefficients of each case (e.g., Houck 1977). Recent empirical research has realized the nonstationary and nonlinear features of time series data and applied cointegration techniques while focusing on short-run asymmetric adjustments (e.g., Goodwin and Holt 1999; Goodwin et al. 2011) or long-run nonlinear price equilibrium (e.g., Gervais 2011; Abbassi et al. 2012) using regime-switching (e.g., threshold, smooth transition, Markov switching) models. Meyer and von Cramon-Taubadel (2004) and Frey and Manera (2007) provided comprehensive reviews of empirical works underlying this literature. Although these modern empirical tools have provided significant convenience in modeling various aspects associated with asymmetric price co-movements, one important aspect was still missing: market co-adjustments under extreme swings. The latest empirical works thus have adopted a novel approach − copulas − to investigate the market dependence when one or more markets is experiencing extreme adjustments or shocks. In addition, copulas also provide information regarding a general structure (a specific copula family and function) and the specific degree of dependence (magnitude of copula parameters) between/among the markets of interest.

Goodwin et al. (2015) was one of the first attempts to introduce the copula approach into the empirical analysis of price transmission. The authors investigated regional North American market integration for a manufactured wood product − oriented strand board. In addition to constant copulas, they also allowed the copula parameters to vary by lagged price differentials to incorporate the regime-switching attribute of price adjustments to deviations from the parity because of transaction costs. The authors directly applied the unconditional copulas, which ignore the potential serial correlation and heteroskedasticity issues often associated with time series data. After Goodwin et al. (2015), several studies investigated price co-movements within the agricultural commodity category using copulas; they usually took the autocorrelation and heteroskedasticity into account and used conditional copulas instead. Zimmer (2015) further proposed a mixed copula model to investigate asymmetric extreme (tail) dependence for three major crops in the U.S. (wheat, corn, and soybeans). Most recently and highly relevant to the current study, Emmanouilides and Fousekis (2015) assessed the structure and degree of market dependencies along the U.S. beef supply chain using copulas. The empirical findings confirmed the existence of asymmetry under extreme swings. Specifically, markets are more likely to co-move when experiencing upswings than downswings. In addition, they found that the farm-retail pair can best be described by an independent copula, which implies no co-movement behavior between the two price adjustments.

Application of the copula method in price co-movement analysis is still young and deserves further investigation and improvement. This study extends the existing application to a generalized autoregressive score (GAS) framework of Creal et al. (2013), which allows the dependence parameters to exhibit a dynamic feature. Our empirical application is to farm-retail price co-movements in the U.S. pork industry, which has undergone substantial variations and changes in the past four decades (technological developments and organizational changes, large market swings such as the hog crisis in 1998/1999, soaring feed costs in recent years, expanding exports, and increasing food services at the retail level). An assumption of constant copulas with time-invariant dependence parameter(s) might be unrealistic. A flexible dynamic framework that allows price co-movements to be (potentially) independent in certain times but depend on each other the rest of the time, with different magnitudes of dependence, is required. Based on the estimated time-varying copula, we further derived the time-varying tail dependencies and simulated the dynamic conditional correlations. The two intuitively easy-to-understand dependence concepts provide us with richer information regarding price transmission for specific periods under changing and complex environments.

2. Empirical Methods

2.1. Copula

A copula is a multivariate distribution whose marginals are all uniform over [0,1]. Given that any continuous random variable can be transformed to become uniform over [0,1] by its probability integral transformation, $U_i \equiv F_i(Y) \sim Unif(0,1)$, copulas can be used to model multivariate dependence structures separately from marginal distributions. For example, a two-dimensional joint distribution, $F(y_1, y_2)$, can be decomposed into two marginal distributions, $F_1(y_1)$ and $F_2(y_2)$, and a two-dimensional copula, C, with the dependence parameter(s), δ:

$$\begin{aligned} &\text{Let}\, Y = [Y_1, Y_2]' \sim F(y_1, y_2), \text{with}\, \mathrm{Y}_1 \sim F_1 \,\text{and}\, \mathrm{Y}_2 \sim F_2 \\ &\text{then}\, \exists\, C\text{: } [0,1]^2 \to [0,1] \\ &\text{s.t.}\, F(y_1, y_2) = C\{F_1(y_1), F_2(y_2); \delta\}\ \forall\, (y_1, y_2) \in R^2 \end{aligned} \tag{1}$$

If $F_1(y_1)$ and $F_2(y_2)$ are continuous, then C is unique; otherwise, C is uniquely determined. Eq.(1) is the well-known Sklar (1959) theorem. An important implication of the Sklar theorem is that we can obtain the joint probability density, which is a product of the marginal probability densities and a copula density function, by taking derivatives of Eq.(1):

$$f(y_1, y_2) = f_1(y_1) f_2(y_2) c\{F_1(y_1), F_2(y_2)\} \tag{2}$$

where $f(y_1, y_2)$ is the joint probability density function (*pdf*) of the two random variables y_1 and y_2, y_i is the *pdf* associated with the marginal distribution F_i, and c is the copula density.

Sometimes, the copula density is called the dependence function because it encodes all the information regarding dependence between the random variables. The copula provides a way to analyze the dependence structure of multivariate distributions without pre-specifying the specific multivariate distribution form. Joe (1997) and Nelsen (2006) are the preferred textbooks. Patton (2012) offered a survey of copula models for financial and economic time series, whereas Genest et al. (2009) studied the issue of goodness-of-fit testing for copulas in an elegant way.

2.2. Conditional copula

In empirical economics, and particularly econometrics using time series data, the random variables of interest being conditioned on pre-determined variables (e.g., past observations) is essential. Most of the time, these variables are more useful than the unconditional joint or marginal distributions themselves. Patton (2006) demonstrated how the existing copula theory could be extended to allow for conditioning variables. He extended the unconditional copula concept of Eq.(1) to a conditional case:

$$\begin{aligned}
&\text{Let} Y \equiv [Y_1, Y_2|M_{t-1}]' \sim F(y_1, y_2|M_{t-1}), \\
&\text{with } \mathrm{Y}_1|M_{t-1} \sim F_1(y_1|M_{t-1}) \text{ and } \mathrm{Y}_2|M_{t-1} \sim F_2(y_2|M_{t-1}) \\
&\text{then} \, \exists \, \mathrm{C} : [0,1]^2 \rightarrow [0,1] \\
&\text{s.t. } F(y_1, y_2|M_{t-1}) = C\{F_1(y_1|M_{t-1}), F_2(y_2|M_{t-1})\} \, \forall \, (y_1, y_2) \in R^2
\end{aligned} \tag{3}$$

where M_{t-1} is the information set at time t. The corresponding density function of Eq.(2) then can be expressed as:

$$f(y_1, y_2|M_{t-1}) = f_1(y_1|M_{t-1}) f_2(y_2|M_{t-1}) c\{F_1(y_1|M_{t-1}), F_2(y_2|M_{t-1})\} \tag{4}$$

2.3. Tail dependence

Tail dependence measures the dependence between the variables in the upper-right quadrant and lower-left quadrant of I^2 (Nelsen 2006). In the current price co-movement application, the upper and lower tail dependencies provide information about the likelihood of the two markets booming and crashing together. The upper tail dependence λ_U is the limit (if it exists) of the conditional probability that Y_2 is greater than the 100h-th percentile of F_2 given that Y_1 is greater than or equal to the 100h-th percentile of F_1 as h approaches 1:

$$\lambda_U = \lim_{h \to 1^-} P\left[Y_2 \geq F_2^{-1}(h) \,\middle|\, Y_1 \geq F_1^{-1}(h)\right]$$

Similarly, the lower tail dependence parameter λ_L is the limit (if it exists) of the conditional probability that Y_2 is less than or equal to the 100h-th percentile of F_2 given that Y_1 is less than or equal to the 100t-th percentile of F_1 as h approaches 0:

$$\lambda_L = \lim_{h \to 0^+} P\left[Y_2 \leq F_2^{-1}(h) \,\middle|\, Y_1 \leq F_1^{-1}(h)\right]$$

Nelsen (2006) demonstrated that tail dependence depends only on the copula of Y_1 and Y_2. The conditional copula version of upper and lower tail dependence can be written as:

$$\begin{aligned} \lambda_U &= \lim_{h \to 1^-} \{1 + 2h + C(h, h|M_{t-1})\}/(1-h) \\ \lambda_L &= \lim_{h \to 0^+} \{C(h, h|M_{t-1})\}/h \end{aligned} \tag{5}$$

If λ_U is in (0,1], two random variables have upper tail dependence; if $\lambda_U = 0$, then they have no upper tail dependence. Similar information can be obtained for λ_L. Furthermore, we can also obtain the time-varying tail dependences (if it exists) by substituting the time-varying copula instead of the constant one into Eq.(5).

A copula function contains all the information about dependence between two random variables. In the price transmission application, if we know the specific copula of the two price changes, then we will be able to obtain all the relevant information regarding the co-movements between the prices. However, numerous copulas exist and each is associated with different dependence attributes. For example, a normal copula allows for a symmetric dependence structure and does not allow for tail dependence; a student t copula allows for joint extreme events in both tails with the same probability. If two markets being more likely to be co-adjusted given an extremely large price increase has been observed; however, price shocks are less likely to be transmitted to one another when shocks are negative. Then either a Joe Clayton copula, which allows asymmetric positive and negative tail dependencies, or a Gumbel copula, which exhibits positive dependence in the upper tail and zero dependence in the lower tail, might be an appropriate choice. For more detailed discussion regarding the dependence attributes associated with different copulas, we recommend Joe (1997) and Nelsen (2006). Given the wide range of copulas, how one chooses the most appropriate copula is a critical issue when being applied. It is thus important to first try a wide range of potential copula models before selecting the most appropriate one. This study estimates the following seven copulas: normal, Clayton, Plackett, Frank, Gumbel, student t, and symmetrized Joe Clayton (SJC) in the first stage and then narrows down based on model selection criteria and goodness-of-fit tests.

2.4. Time-varying copula

Copula functions provide rich information about dependency between random variables. However, a constant copula assumes a fixed structure and degree of dependence and thus is unable to integrate constantly changing conditions. The time-varying attributes may come from factors such as structural change in the industry, new policy interventions, and changing market participants behaviors. This type of dynamic can be modeled by allowing either the copula function, when the whole dependence structure (e.g., a switch from a positive to a negative relationship) changes, or the copula parameter(s), if only the degree of dependence varies, to become time-variant. A comprehensive review of time-varying copula applications focusing on specification and estimation can be found in Manner and Reznikova (2012).

Okimoto (2008) and Rodriguez (2007) studied regime-switching models for the conditional copula and allowed the functional form of the copula to vary through time. Patton (2006) and Creal et al. (2013), in contrast, considered specifications of time-varying copulas where the copula functional form was fixed and its parameters were allowed to vary through time as a function of past information, similar to the generalized autoregressive conditional heteroskedasticity (GARCH) specification for volatility. We followed this latter line of research and adopted the generalized autoregressive score (GAS) framework developed by Creal et al. (2013), which specifies the copula parameter (θ_t) as a function of the lagged parameter and a scaled standardized score of the copula log-likelihood. To deal with parameters that are restricted to a particular range (e.g., correlations must lie inside [-1,1]), Creal et al. (2013) suggest applying a strictly increasing transformation, g, to the copula parameter and models the evolution of the transformed parameter, denoted as φ_t:

$$\begin{aligned} \varphi_t &= g(\theta_t) \Leftrightarrow \theta_t = g^{-1}(\varphi_t) \\ \varphi_t &= \omega + \beta \varphi_{t-1} + \alpha I_{t-1}^{-1/2} \nabla_{t-1} \end{aligned} \tag{6}$$

where $\nabla_t = \frac{\partial}{\partial \theta} \log c(u_{1t}, u_{2t}; \theta_t)$ and I_t is the Hessian matrix. Thus, the present value of the copula parameter is a function of a constant, the past value and the score of the copula-likelihood $I_{t-1}^{-1/2} \nabla_{t-1}$. Creal et al. (2013) motivate the GAS framework by demonstrating that it nests a variety of popular existing models such as the GARCH model of Bollerslev (1986) and its variants and Poisson counts model of Davis et al. (2003).

2.5. *Copula-based dynamic conditional correlations*

The Pearson's linear correlation coefficient is by far the most familiar dependence concept; it can be obtained by dividing the covariance of the two variables by the product of their standard deviations. Early work in dependence studies calculated simple constant correlation coefficients based on sample covariance and variances. Later simple methods, such as rolling historical correlations and exponential smoothing techniques, were widely adopted to reveal the time-varying market dependence. In modern empirical work, more complex methods, such as the variety of multivariate GARCH models have been extensively adopted to investigate the conditional correlations between two random variables.[1] Although these modern GARCH models have provided considerable flexibility in deriving dynamic conditional variances and covariances, and thus the dynamic conditional correlations (DCC), they are obtained based on a restrictive (often unrealistic) assumption: The multivariate distributions follow multivariate normal or t distributions. Though these two distributions and their variants are usually sufficient to model univariate price changes, there is no justification for suggesting that all of the markets under consideration come from the same distributional family. Even if the same distribution applies to both markets, the distributional parameters (e.g., degree of freedom) need not be the same.

This study thus used a time-varying copula approach to obtain the DCC, allowing marginal distribution of each price change series to come from different families or the same family with different distributional parameters. Specifically, a numerical integration or a simulation approach can be used to derive the DCC (Patton 2013). One can write the dynamic conditional correlation of the two variables as:

$$\begin{aligned}\rho_{t-1} &= Corr_{t-1}(Y_1, Y_2|M_{t-1}) = Corr_{t-1}(\varepsilon_{1t-1}, \varepsilon_{2t-1}) \\ &= E_{t-1}(\varepsilon_{1t-1}\varepsilon_{2t-1}), \text{ where} \varepsilon_{it-1} \sim F_i(0,1) \\ &= E_{t-1}[{F_1}^{-1}(U_1){F_2}^{-1}(U_2)]\end{aligned} \tag{7}$$

Eq.(7) usually cannot be obtained analytically; however, it can be solved by using the two-dimensional numerical integration:

$$E_{t-1}[{F_1}^{-1}(U_1){F_2}^{-1}(U_2)] = \int_0^1 \int_0^1 {F_1}^{-1}(u_1){F_2}^{-1}(u_2)c\{u_1, u_2; \delta_t\}du_1 du_2$$

where c is the probability density function of the time-varying copula. Another approach is to rely on the simulation technique:

$$E_{t-1}[{F_1}^{-1}(U_1){F_2}^{-1}(U_2)] \approx \frac{1}{S}\sum_{s=1}^{S} F_1^{-1}(u_1^{(s)})F_2^{-1}(u_2^{(s)}) \tag{8}$$

where $(u_1^{(s)}, u_2^{(s)}) \sim iid\ C(\delta_t)$ and S is the number of simulation. When the copula is time-varying, the simulation needs to be conducted for each month if exploiting monthly data, as each month will have a different value for the copula parameter.

2.6. Copula estimation

An intuitive approach to fit a multivariate distribution is through the maximum likelihood estimation of the full multivariate distribution. Given the multivariate density function of Eq.(4), provided that F_1 and F_2 are differentiable and F and C are twice differentiable, the log-likelihood expression can be fully specified:

$$l(\vartheta) = \sum_{t=1}^{T}\sum_{i=1}^{2} \ln f_i(y_i|M_{t-1};\vartheta_i) + \sum_{t=1}^{T} \ln c\left(F_1(Y_1|M_{t-1};\vartheta_1), F_2(Y_2|M_{t-1};\vartheta_2);\delta_t\right)$$

where $\vartheta = [\vartheta_1', \vartheta_2', \delta_t']$ represents the parameter vectors associated with the marginal distributions and the copula function. Although estimating all coefficients simultaneously yields the most efficient estimates, the large number of parameters can make numerical maximization of the likelihood function difficult.

A popular alternative is the so-called canonical maximum likelihood method, as applied in Emmanouilides and Fousekis (2015). This method avoids the problem of finding and estimating specific parametric models for the marginals by using the empirical distribution functions instead. The asymptotic distribution of this estimator has been studied by Genest et al. (1995) for *i.i.d.* data and by Chen and Fan (2006) for time series data. Though convenient, this method only applies to constant copulas. If a copula is time-varying, the inference methods for the copula parameters, obtained from the canonical maximum likelihood method, are not yet available in the econometrics literature (see Patton 2013 for a detailed discussion).

Another flexible option (probably the most commonly used method) is a sequential two-stage maximum likelihood (TSML) method in which the marginals are estimated in the first stage and then the estimated parameters are substituted into the copula function to estimate the dependence parameter(s). This method allows for consistent estimates of the copula parameters and can avoid the "curse of dimensionality" by separately estimating the one-dimensional marginal distributions and the multi-dimensional copula. Furthermore, inferences based on the parametric distributions of marginals and the time-varying copula can be drawn using bootstrapping or simulation methods. This study hence adopted the TSML method.

3. The U.S. Pork Industry and Data

3.1. The U.S. Pork Industry

Enlarged concentration at the retail, processing, and farm levels has brought increased attention to patterns in farm-to-retail price transmission. U.S. pork production has experienced substantial changes in the past half century. Until the 1980s, the industry primarily comprised numerous small farms that had integrated farrow-to-finish operations and focused on selling through open markets for domestic consumption. Since the 1990s, however, the industry has undergone extensive changes to become an industry driven by fewer and larger producers who are finishing specialists and who sell contracted production to both global and domestic markets. Organizational change in hog production, particularly the widespread use of contracting, has enabled individual farmers to grow by specializing in a single phase of production. In 1993, nearly 90% of U.S. pork producers sold hogs on the open market. In contrast, more than 80% of U.S. hog production today is under contract to processors (Key and McBride 2007). Technological innovations have also been a driving force behind the changes and have contributed to substantial increases in hog farm productivity.

These changes have resulted in substantial gains in efficiency and lower production costs for hog farms. Productivity gains contribute to about a 30% reduction in the price of hogs at the farm gate (Key and McBride 2007), which creates the potential for substantial benefits to consumers (Haley 2004). However, there are concerns that consumers have not benefited as much as expected since processors and retailers do not pass on the associated cost reductions. Previous research in the U.S. meat industry (e.g., Goodwin and Holt 1999; Goodwin and Harper 2000; Gervais 2011) has also confirmed a distinct asymmetry in retail responses to farm-level price changes. In particular, upward movements in farm prices are followed by retail price adjustments more quickly and substantially than downward farm-level price movements. In response to this concern, we investigated the dynamic co-movements between farm and retail price changes. The results provide useful information regarding market integration and efficiency for the U.S. pork industry along the vertical supply chain.

3.2. Data

Monthly data on hog (farm) and pork (retail) prices from January 1970 through March 2013 (528 observations) come from the United States Department of Agriculture Economic Research Service (USDA/ERS 2014). The farm price is the AMS 51%-52% base lean-hog price. The retail price is a weighted average of the retail prices for specific pork cuts based on the values reported by the Bureau of Labor Statistics. To make the price comparisons easier, ERS transforms the farm price into a retail-weight equivalent.[2] While the ERS prices are meant to capture the value of a consistent standard animal over time, inflation makes it difficult to compare 1970s values with more current values. We thus deflate the nominal prices to the real price levels using CPI (2010=100). Figure 1 displays the logarithmic (log_{10}) farm and retail prices and price changes. The farm price exhibits a decreasing trend until the mid-2000s. The relatively constant (not declining) price in recent years may be a result of soaring crop prices as feed represents 65% to 70% of the cost of producing a pig, while corn (or a close substitute, such as grain sorghum or barley) makes up about 60% of total feed costs (Key and McBride 2007). The farm price reached a historical minimum in December

1998, the so-called "hog crisis." A rough visual inspection also indicates that farm price changes are more volatile than retail price changes. Summary statistics for price and price change series are presented in Table 1.

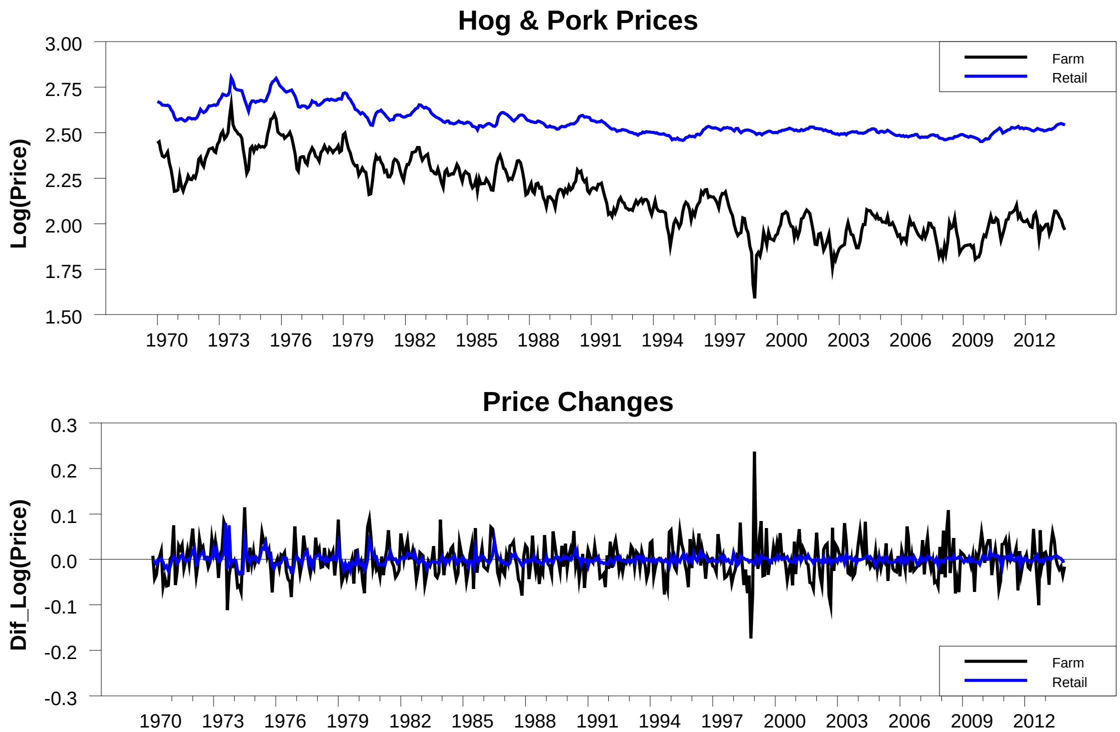

Fig. 1. Monthly Pork/Hog Real Prices and Price Changes: 1970- 2013

4. Empirical Procedures and Results

The empirical procedure for the time-varying copula application in the price transmission analysis can be divided into four steps: (1) Model the conditional marginal distributions for each price adjustment series, (2) estimate the selected constant copula models based on the results obtained from the first step, (3) test and estimate the time-varying copula, and (4) derive tail dependencies, simulate the dynamic conditional correlations based on the time-varying copula parameters, and link these results to price co-movements/transmission implications.

Table 1: Summary Statistics for Log Prices and Price Changes

	P^F	P^R	ΔP^F	ΔP^R
Mean	2.154	2.559	-0.001	0.000
Std error	0.193	0.075	0.038	0.01
Skewness	0.112	1.001	0.402	1.417
Kurtosis	2.164	3.291	6.31	13.378
ADF test statistics	-1.813	-2.442	-6.114***	-11.816***
(Lags)	(12)	(1)	(11)	(2)
N=528				

Note: The critical values of the ADF test at 1%, 5%, and 10% are -2.865, -2.569, and -3.440, respectively. Lags in ADF tests are based on Bayesian Information Criterion (BIC). *** denotes rejection of null hypothesis at the 1% significance level.

4.1. Model the marginal distributions

We denote price changes, p^R and p^F, as the log-differenced retail and farm prices (i.e., $p_t^i = \log P_t^i - \log P_{t-1}^i$). The first step is to model the conditional marginal distribution for each series of price changes, which is equivalent to modeling the distribution of standardized residuals. Before proceeding to the marginal distribution modeling, we first must model the conditional mean and variance to obtain the standardized residuals. Similar to Zimmer (2015) and Emmanouilides and Fousekis (2015), we specified the conditional mean and variance using an autoregressive and GJR-GARCH (Glosten et al. 1993) framework. In addition, we included the cross-equation (CE) effects in the conditional mean equation when applied, and we allowed the volatility to have potentially asymmetric attributes based on the direction of shocks. The mean and variance equation for the retail price changes can be expressed as:[3]

$$
\begin{aligned}
&p_t^R = \alpha_1 + \sum_{i=1} \beta_{1i} p_{t-i}^R + \sum_{j=1} \gamma_{1j} p_{t-j}^F + \sqrt{h_{1t}}\varepsilon_{1t}, \quad \text{and} \quad \varepsilon_{1t} \sim F_1(0,1), \\
&h_{1t} = c_1 + a_1 u_{1t-1}^2 + b_1 h_{t-1} + dIu_{1t-1}^2, \\
&\text{where } I = 1 \text{ if } u_{1t-1} < 0, \quad \text{and } I = 0 \text{ otherwise.}
\end{aligned} \tag{9}
$$

From Eq.(3), we can obtain the standardized residuals as:

$$\hat{\varepsilon}_{1t} = \left[{p_t}^R - \left(\hat{\alpha}_1 + \sum_{i=1} \hat{\beta}_{1i} p^R_{t-i} + \sum_{j=1} \hat{\gamma}_{1j} p^F_{t-j} \right) \right] \Big/ h^{1/2}_{1t} \tag{10}$$

A similar model specification can be applied to the series of farm price changes. Many choices are possible for modeling marginal distributions. Zimmer (2015) adopted a normal distribution and Emmanouilides and Fousekis (2015) used nonparametric empirical distributions. As discussed in the copula estimation section, the inference methods for the copula parameters are not yet available in the literature for adopting nonparametric distribution for marginals, meanwhile allowing the dependence parameters to be time-varying. Furthermore, since fat-tail and skewness are two common characteristics associated with price data and the unconditional price change series also exhibits these attributes (see Table 1), we thus used the skewed t distribution developed by Hansen (1994), which takes the skewness and fat-tail into account. This skewed t specification also incorporates normal (when the degree of freedom is close to infinite) and standardized student t (when the skewness parameter equals zero) as two special cases, which therefore provides a great deal of flexibility in modeling the marginal distributions.

The results of the conditional means and variances using Eq.(9) are presented in Table 2. Lag selections are based on BIC. The optimal models were found to be an AR(3)-GARCH (1,1) for retail price changes and an AR(2)-asymmetric GARCH (1,1) for farm price changes. Testing for the significance of three lags of the other series, based on the optimal AR-GARCH specification, we found no evidence of significant cross-equation effects in the conditional mean for the farm series. For the retail case, we found one lag CE effect. The results indicated that retail price responds to farm-level price adjustments but not vice versa. The unidirectional causality from the farm to downstream market levels is consistent with intuition and findings from previous studies on U.S. meat price transmission (e.g., Heien 1980; Goodwin and Holt 1999; Gervais 2011). The asymmetric effect from negative market shocks has been identified at the farm-level conditional volatility. For the retail case, the conditional volatility reacts to negative price shocks in the same way. Figure 2 displays the estimated volatility for the two markets. Overall, the farm-level price volatility is larger and more volatile than the retail price volatility. For the retail price, changes seem to be more volatile before the late-1980s and have been relatively stable since then. Even during the hog crisis and the world food crisis, we do not observe any unusual spikes. However, we do not observe a decreasing volatility trend for farm price changes; the swings are extremely large during the hog crisis and relatively high in 2007/2008 when world food prices soared.

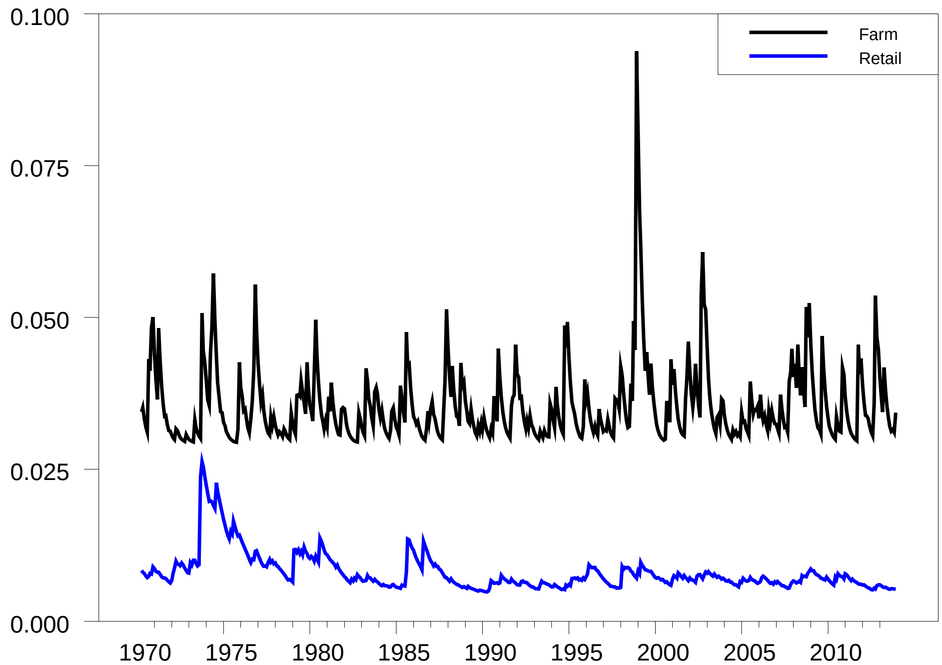

Fig. 2. Conditional Volatility of Pork/Hog Price Changes

To test the appropriateness of the skewed t distribution, we conducted the Kolmogorov-Smirnov (KS) and Cramer-von Mises (CvM) goodness-of-fit (GoF) tests:

$$KS_i = \max_t \left| \hat{U}_{i,(t)} - \tfrac{t}{T} \right|$$
$$CvM_i = \sum_{t=1}^{T} \left(\hat{U}_{i,(t)} - \tfrac{t}{T} \right)^2$$

where $\hat{U}_{i,(t)} = \hat{F}_{skewed-t}(\hat{\varepsilon}_{it}; \hat{\nu}_i, \hat{\lambda}_i)$ is the t^{th} largest value of $\left\{ \hat{U}_{i,j} \right\}_{j=1}^{T}$.

The KS and CvM test results from modeling the marginal distribution of standardized residuals using skewed t distribution are presented in Table 3. For the farm case, the skewness

Table 2: Estimates of Mean and Variance Equations

Farm			Retail		
Variable	Coeff	Std Error	Variable	Coeff	Std Error
Constant	-0.002	0.002	Constant	0.000	0.000
p^F_{t-1}	0.197***	0.051	p^F_{t-1}	0.080***	0.009
p^F_{t-2}	-0.100***	0.045	p^R_{t-1}	0.169***	0.047
c	0.000***	0.000	p^R_{t-2}	0.218***	0.045
a	0.004	0.03	p^R_{t-3}	-0.087***	0.039
b	0.683***	0.115	c	0.000***	0.000
d	0.237***	0.076	a	0.548***	0.112
			b	0.310***	0.087
Log-Likelihood		1009.077			1813.25

Note: ***, **, and * denote rejection of the null hypothesis at the 1%, 5%, and 10% significance levels, respectively.

parameter is small, which indicates symmetry and that a standard t distribution might be sufficient. For the standardized residuals from the retail price changes, the positive skewness parameter means the distributions are asymmetric and positively skewed. Both KS and CvM results fail to reject the null hypothesis that skew t is an appropriate distribution for modeling the two standardized residuals.

Table 3: Skewed t Density Parameters and Tests

	Farm	Retail
Degree of Freedom (ν)	15.288	6.025
Skewness (λ)	0.033	0.183
KS test p-value	0.39	0.29
CvM test p-value	0.30	0.35

4.2. Constant copulas

The estimated results from seven constant copulas are presented in the appendix (Table A1). Based on the log-likelihood values, we picked the best-fitted cupola, the Gumbel copula, for further potential dynamics investigation. The Gumbel copula allows for upper tail dependence and no lower dependence, which indicates that price co-movements are likely to occur during market upturns but not downturns. This provides evidence for the concern that in the meat industry, large upward price adjustments at the farm level are more likely to be matched at the retail level, while large downward farm-level price changes are not likely to be transmitted to the retail level.

4.3. The time-varying copula

Before estimating the time-varying conditional copula model, it is informative to test for the presence of time-variant dependence. We adopted the test proposed by Patton (2013), which is similar to the ARCH-LM test for time-varying volatility proposed by Engle (1982). The test looks for autocorrelation in a measure of dependence, captured by an autoregressive-type model. Consider the following regression:

$$U_{1t}U_{2t} = \theta_0 + \sum_{i=1}^{p} \theta_i U_{1t-i}U_{2t-i} + e_t$$

where $U_{it} = F_{skewed-t}(\hat{\varepsilon}_{it}; \hat{\nu}_i, \hat{\lambda}_i)$ is obtained from the marginal distributions. Under the null of a constant conditional copula, we should find $\theta_i = 0$, $\forall i \geq 1$, which can be tested by forming the statistic:

$$\hat{\mathrm{A}}_p = \hat{\theta}' R' (R\hat{V}R')^{-1} R\hat{\theta} \tag{11}$$

where $\hat{\theta} = [\theta_1, \theta_2, ..., \theta_p]$, $R = \left[0_{p\times 1} \vdots I_p\right]$, and using the OLS estimate of the covariance matrix for $\hat{V}$. The p-value for this test statistic can be obtained by bootstrapping.[4] The test (see Table 4) rejects the constant rank correlation hypothesis and supports the time-varying hypothesis for orders 1 and 2, and we thus move on to estimating the time-varying Gumbel copula.

The Gumbel copula parameter is required to be greater than 1, and the function $\theta_t = 1 + \exp(\varphi_t)$ is used to ensure this. We then proceed to the time-varying copula estimation with the model specification of Eq.(6). The estimate results (see Table 5) indicate that the

Table 4: Tests for Time-Varying Dependence

	p-value
AR(1)	0.005
AR(2)	0.012
AR(3)	0.213

Note: The p-value is obtained from bootstrapping (replicate=10,000).

time-varying copula has a much higher log-likelihood value than in corresponding constant copula cases. The two evolution parameters (α and β) are both significant. GoF tests that use the empirical copula of the data rely on the assumption that the true conditional copula is constant and so are inappropriate for time-varying copula models. Thus, for the time-varying copula, the Rosenblatt transformed KS and CvM tests were adopted, as proposed by Rèmillard (2010). In this approach, the data are first transformed so that, if the model is correct, the data are independent $u(0,1)$ random variables, and then KS and CvM tests are applied to the transformed data. The Rosenblatt transformed KS and CvM test results are presented in Table 6. Neither test could reject the null hypothesis that the estimated time-varying copula and the underlying unknown empirical copula are from the same family.

Table 5: Estimated Time-Varying Gumbel Copula Parameters

Parameter	Estimate	Std Error
ω	-0.007	0.065
α	0.064***	0.011
β	0.990***	0.102
Log-Likelihood	52.421	
N=525		

Note: ***, **, and * denote rejection of the null hypothesis at the 1%, 5%, and 10% significance levels, respectively.

Table 6: Goodness-of-Fit Tests for the Time-Varying Gumbel Copula

	KS_R	CvM_R
p-value	0.35	0.2

Note: KS_R and CvM_R refer to the Rosenblatt transformed KS and CvM tests.

4.4. Time-varying tail dependence

Direct interpretation of the time-varying copula parameters is not straightforward. We present the associated time-varying tail dependence and the simulated conditional correlations to reveal the dynamic dependence attributes. Figure 3 displays the upper tail dependence between two conditional series of price changes. The results indicate that dependence between two markets, when experiencing large upturns, displayed an increasing trend in the 1970s, but decreased in the 1980s and reached the historical lowest level during the hog crisis in 1998/1999; it has remained relatively low since then, with a temporary minor increase in 2002-2004. The average upper tail dependencies in the 1970s and 1980s are about 0.40 and 0.35, which implies that the probabilities of a large upward change at the farm level being associated with a corresponding adjustment at the retail level are around 0.40 (for the 1970s) and 0.35 (for the 1980s). In the 1990s, the average tail dependence dropped to 0.17. After 2000 and until the end of 2013, the mean upper tail dependence dropped further to about 0.12.

During the hog crisis, upper tail dependence reached a low level of 0.13.[5] The reason we observe such a low value is related to market responses to the previous adjustments. During the crisis, the farm-level hog price decreased dramatically by 61% while the decline in the retail-level pork price was trivial (less than 2%); this is consistent with the zero lower tail dependence. Right after the crisis, the farm-level price increased significantly as it went back to the normal market level, while the retail level only made minor upward adjustments because it had not declined much in the first place. Following the previous large decreases, the farm-level price increases as it goes back to its long-term trend. The retail price then

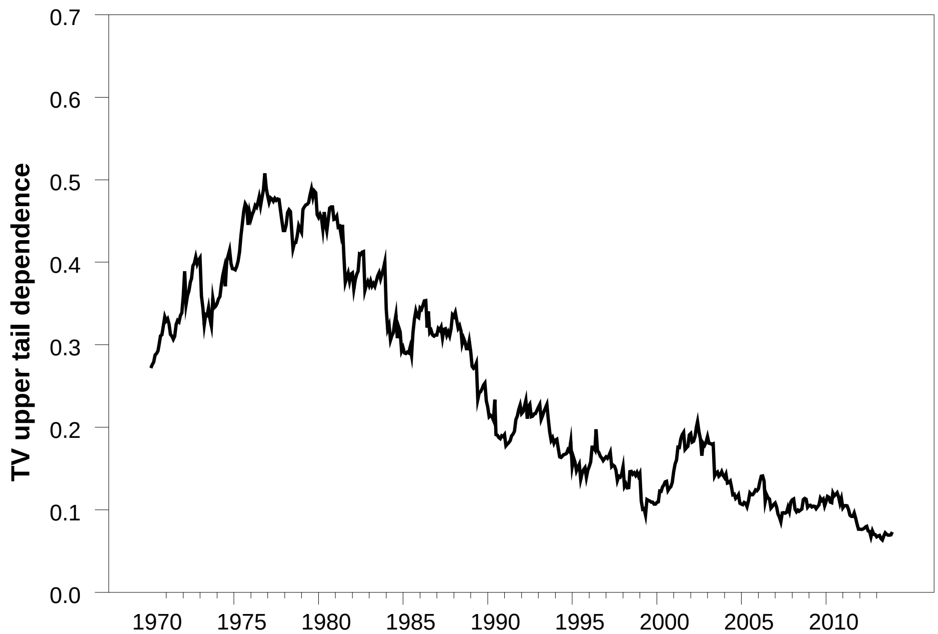

Fig. 3. Time-Varying Upper Tail Dependences

increases as a follow-up to farm-level adjustments, but with a much smaller magnitude. That explains why we observed small upper tail dependencies. Since the causality in price adjustments is unidirectional (refer to Table 2), the co-movement pattern at the tails implies that huge positive price adjustments at the farm level are likely to be transmitted to the retail level with a higher intensity in the early period than today.

4.5. Copula-based dynamic conditional correlations

Finally, the results of the dynamic conditional correlations from the time-varying Gumbel are plotted in Figure 4, together with the upper tail dependence for comparison purposes. The conditional correlations decrease as time goes by (as real prices decrease). The general trend of DCC is very similar to the upper tail dependence. In the 1970s, the two price adjustments were highly dependent on each other, with an average mean DCC of 0.54. The DCC exhibited an increasing trend in the 1970s; however, around 1980 the correlations started declining and kept decreasing for the next 20 years (with a mean value of 0.31) and reached the historical low (0.08) in February 1999 during the hog crisis. Over the past 43 years, the lowest DCC (with a mean value of 0.07) corresponds to the most recent two years (2012-2013). The results indicated that the degree of market dependence along the farm-to-retail supply chain has been decreasing over the study period and currently has reached an extremely low level. As we discussed, during the past several decades, the industry has experienced dramatic changes. Productivity improved substantially because of organizational rearrangements, technological improvements, and economies of scale. From about 1991 to 2003, the real farm price decreased, but the retail price did not match that downward move and only made minor adjustments occasionally. Overall, the retail price did not display a corresponding downward trend and remained relatively stable and constant. That is one primary reason that we observe decreasing correlations between the two price adjustments.

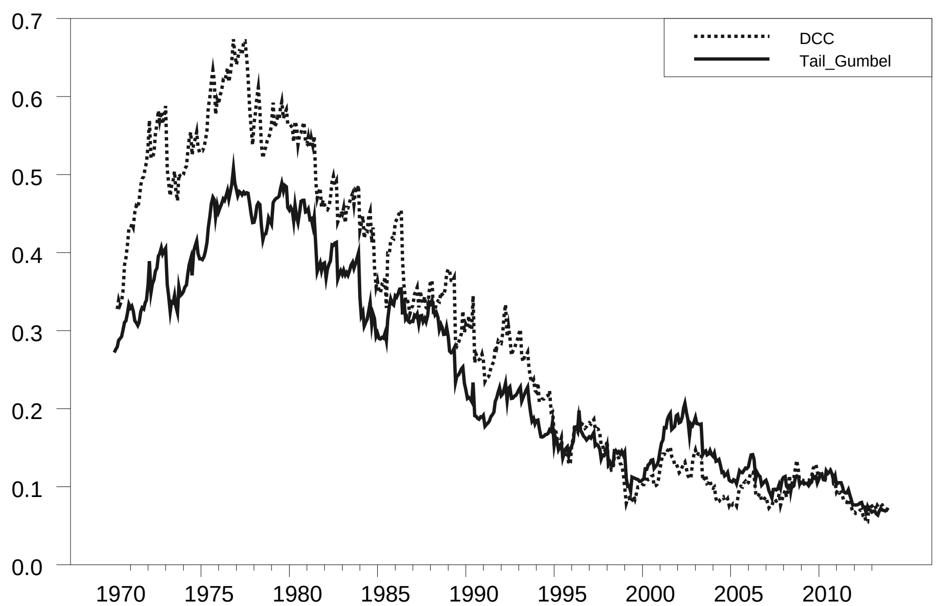

Fig. 4. Dynamic Conditional Correlations and Upper Tail Dependences

Another interesting finding revealed a changing relationship between the DCC and the upper tail dependencies. Before the hog crisis, the DCC is usually higher than the upper tail dependencies. In other words, the general dependence between two market adjustments is higher than the probability of co-movements under large positive adjustments. However, as Figure 4 shows, after the hog crisis, both DCC and tail dependence become extremely low; tail dependence moves either higher than or quite close to DCC. This interesting result shows that the hog crisis may have served as a threshold before which the overall dependence between farm and retail price adjustments was higher than the dependence under the extreme (positive) price adjustments. However, after the hog crisis, the overall dependence is close to (or sometimes even lower than) the probability of market co-adjustments under huge upturns.

In summary, the farm and retail price adjustments are highly dependent between 1970 and the mid-1980s and have become less reliant since then. In particular, after the hog crisis in 1998/1999, the correlations dropped to as low as about 0.1. From about 1993 to 2003, the farm-level price decreased, but the retail price did not make corresponding downward adjustments in general and stayed relatively constant. From around 2006 to the present, the farm price stopped the downward trend and remained relatively constant (though still quite volatile), probably because of the high feed prices in recent years. Meanwhile, the retail price was still quite constant without any significant upward adjustments. That is consistent with the findings of decreasing conditional correlations. Furthermore, results indicated that retail price adjustments exhibit downward stickiness. When real prices are high, retail prices are more likely to adjust according to the farm-level price changes (thus resulting in larger DCC). However, when real prices are low, they are less likely to match the farm-level adjustments. Figure 5 depicts the scatter plots of real (farm and retail) prices and the dynamic conditional correlations. A positive relationship can be observed between real prices and the DCC, which also confirms that price transmission decreases as the real prices decrease. This finding is consistent with previous research (e.g., Gervais 2011) that also found downward price stickiness in the U.S. hog/pork industry using the nonlinear cointegrating regression model. In addition to the potential market power enjoyed by the retailers, another possible explanation might be the menu costs associated with retail price adjustments.

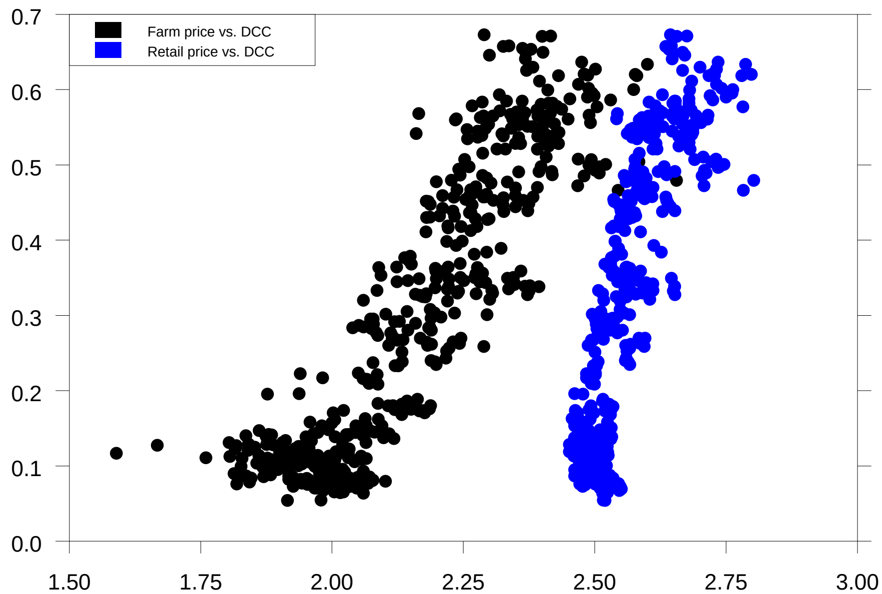

Fig. 5. Scatter Plot of Real Prices and the Dynamic Conditional Correlations

5. Conclusions

We developed a time-varying copula-based price co-movement framework that can be used to investigate the dynamic conditional correlations and tail dependencies. Our empirical application is to farm-retail price co-movements for the U.S. pork industry. According to the simulated DCC, the degree of market dependence between farm and retail level along the pork supply chain decreased considerably from 1970 to 2013. The declining pattern of the farm and retail price co-movements may indicate market inefficiency along the pork production vertical chain. Combined with the reality that productivity gains contributed to a more than 30% reduction in the price of hogs at the farm gate, the gains and benefits are not being passed through to consumers. One potential reason is that food retailers bargaining power has increased significantly in recent years (Emmanouilides and Fousekis 2015). Results also showed that retail price adjustments exhibit downward stickiness, with one potential explanation being menu costs associated with price changes.

In addition to DCC, zero lower tail dependence and positive upper tail dependence revealed asymmetric co-movements during extreme market adjustments. In particular, the probability that retail price would make a corresponding adjustment, given a large-scale downward adjustment in the farm level, is zero. On the other hand, positive upper tail dependence signifies that retail prices do react to large positive adjustments at the farm level; however, the probability declines significantly as the real price decreases.

Finally, the contribution of this paper lies predominantly in promoting a methodological framework which to date has seen little empirical application in the context of price transmission. In addition to being an important measurement of price co-movements, correlations are also essential inputs for financial and risk management tasks. For example, hedge designs require estimates of the correlation between the returns of assets under contract. A forecast of future correlations is the basis of any option pricing formula. The copula-based approach presented in this paper thus provides a flexible and useful extension and generalization of conventional approaches for estimating DCC, which can be applied to other relevant fields.

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Notes

[1]For example, the CCC-GARCH model of Bollerslev (1990), the DCC-GARCH model of Engle (1982), and the AG-DCC-GARCH model of Cappiello et al. (2006).

[2]ERS uses a conversion factor of 1.869 pounds of 51%-52% lean hog to produce a pound of "standard" pork.

[3]Note that we do not include the error term in the mean equation specification. We conducted the Johansens trace test for cointegration, and results suggested that one linear long-run relationship exists for each pair of price adjustments. However, using the same data source, Gervais (2011) found evidence of nonlinear cointegrating relationships. The two sets of residuals obtained from different cointegrating relationships are quite different; we thus dismiss the idea of including the error terms in the model specification. It is a common practice to not include an error term in the mean equation when using the copula method (e.g., see Zimmer 2015; Emmanouilides and Fousekis 2015). However, future studies interested in investigating both short-run and long-run transmission may find it helpful to conduct a cointegration test first and include the error term (if it exists) in the mean equation to avoid potential model misspecification.

[4]Specifically, we first randomly drew rows, with replacement, from the Tx2 matrix of standardized residuals and obtained a bootstrap sample with observations of T=525; then we estimated the statistics of Eq.(11) for the bootstrap sample; we repeated the procedure 10,000 times and used the bootstrapped distribution of estimated statistics to obtain the p-value for the estimated statistics.

[5]We define the hog crisis here as the period from December 1998 through April 1999.

Appendix A.

Table A1. Constant Copulas

	θ_1	θ_2	**Log-likelihood**
Gumbel	1.285	-	38.1
Normal	0.357	-	35.8
Clayton	0.414	-	23.8
Plackett	3.044	-	35.0
Frank	2.269	-	33.7
Student t	0.363	0.088	37.6
SJC	0.126	0.222	37.1

Note: Estimate results from constant copulas. SJC refers to the symmetrized Joe Clayton copula. Parameters 1 and 2 for the student t copula are tail dependence and degree of freedom, respectively; Parameters 1 and 2 for the SJC copula are upper and lower tail dependence, respectively.